The Village School's
AP Calculus AB Summer Review
& Real–World Activities

Welcome to AP Calculus AB at The Village School. This packet is comprised of important concepts necessary for success in AP Calculus AB. Completion of this packet is mandatory for all AP Calculus AB students and must be done in pencil. As you complete this packet, show all steps used to arrive at your final answer. There are two parts to this packet:

Part 1 is comprised of mathematical practice problems to keep your math skills sharp. The exercises are direct and do not take much time. The packet will be graded on completion, so keep trying problems, even if you have trouble.

Part 2 is comprised of two STEAM activities for you to explore. Show your work and answers on separate paper. **Warning:** Part 2 will take some time, so make sure that you do it as soon as you can.

Part 1: Mathematical practice problems to keep your math skills sharp.

LINES:

Write a linear equation for the function.

1. With slope $-\frac{4}{5}$ and *y*-intercept -2.
2. With slope $\frac{2}{3}$ through $(5,-1)$.
3. Through $(1,5)$ and $(3,-1)$.
4. With *x*-intercept 2 and *y*-intercept 4.
5. Parallel to $4x-3y=7$ through $(6,-2)$.
6. Perpendicular to $4x-3y=7$ through $(5,1)$.

FUNCTIONS:

Evaluate and simplify each function as directed.

7. $f(x)=4x-7$
   a. $f(-3)$
   b. $f(a)$
   c. $f(x+h)$
   d. $\frac{f(x+h)-f(x)}{h}$
8. $f(x)=2x^2-3x-4$
   a. $f(3)$
   b. $f(4y)$
   c. $f(x+h)$
   d. $\frac{f(x+h)-f(x)}{h}$
9. $g(x)=\frac{2}{x+3}$
   a. $f(-3)$
   b. $f(0)$
   c. $f(x+h)$
   d. $\frac{f(x+h)-f(x)}{h}$
10. $y(x)=\sqrt{x-5}$
    a. $f(0)$
    b. $f(a-3)$
    c. $f(x+h)$
    d. $\frac{f(x+h)-f(x)}{h}$

Find the domain and range for each of the following functions.
Write your answers in interval notation.

11. $f(x)=(x-3)^2-4$
12. $y=\sqrt{x-6}+3$
13. $g(x)=2|x-7|-1$
14. $f(x)=\sqrt{25-9x^2}$
15. $f(x)=\frac{2x+3}{x-1}$
16. $f(x)=\frac{x-3}{x^2-9}$
17. $f(x)=\frac{\sqrt{x}}{x-4}$

Find the inverse of each function. Verify your inverse by computing $(f \circ g)(x)$ ***or*** $(g \circ f)(x)$ ***which will equal*** x ***if they are indeed inverses of each other.***

18. $f(x) = 4 - 10x$

19. $g(x) = 5x^3 + 2$

20. $f(x) = 3 - \sqrt{x}$

21. $f(x) = \frac{2x+3}{x-1}$

Factor each of the following expressions.

22. $25x^2 - 16y^2$

23. $4x^2 + 12x + 9$

24. $12x^5 - 10x^4 - 8x^3$

25. $x^7 + 6x^4 - 16x$

26. $6x^3y - 26x^2y^2 + 8xy^3$

27. $x^6 - 1$

Solve each of the following equations.

28. $5x + 9 = 2(x + 2)$

29. $y^2 = 5y + 14$

30. $3x^2 - 6x - 7 = 0$

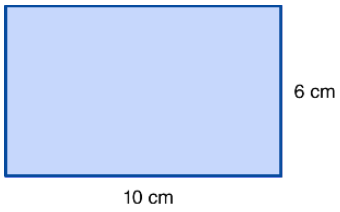
31. $x^3 + 3x^2 = 5x + 15$

GEOMETRY:

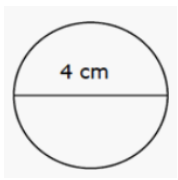
Find the specified values. Look up any formulas that you do not remember.

32. Find the length and width of a rectangle with area, $A = 36\ cm^2$, and perimeter, $P = 26\ cm$.

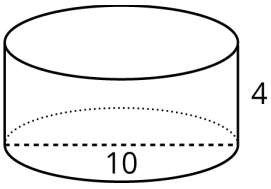
33. Find the area and perimeter.



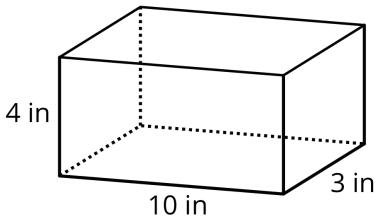
34. Find the area and circumference.



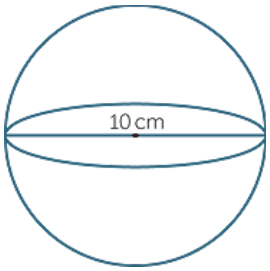
35. Find the volume and surface area.

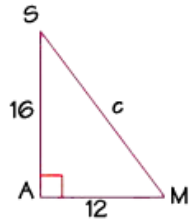


36. Find the volume and surface area.



37. Find the volume and surface area.





38. Find the hypotenuse. Then find $\sin M$, $\cos M$, $\tan M$, $\sec M$, $\csc M$, and $\cot M$.

Sketch each of the following functions and equations, <u>without</u> the use of a graphing calculator or Desmos.

39. $3x + 2y = 18$

40. $f(x) = 2\,|x + 3| - 5$

41. $y = \sqrt{x - 6} + 3$

42. $f(x) = 2x^2 + 8x + 3$

43. $g(x) = (x + 5)^2 + 3$

44. $f(x) = \ln x$

45. $f(x) = 3^x$

46. $f(x) = 3^{-x}$

47. $y = 2\sin\left(2x - \frac{\pi}{3}\right) - 1$

48. $x^2 - 6x + y^2 + 8y + 24 = 0$

49. $\frac{(x+4)^2}{25} + \frac{(y+2)^2}{9} = 1$

Find the limits.

50. $\lim_{x \to 8} \sqrt{x + 8}$

51. $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

52. $\lim_{x \to 5} \frac{x^2-25}{x^2-4x-5}$

53. $\lim_{x \to \infty} \frac{x^2-25}{x^2-4x-5}$

54. $\lim_{x \to \infty} \frac{x+2}{x^2-4x+3}$

Part 2: STEAM activities for you to explore

STEAM Activities

Directions:

On the pages that follow you will find two STEAM activities. You must complete BOTH of the activities and bring to school in August.

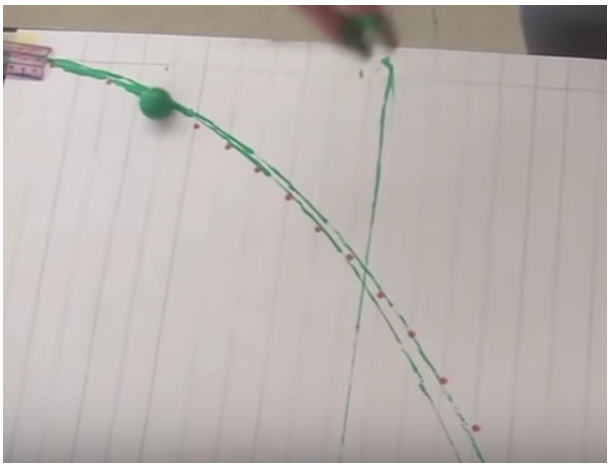
1. The first activity requires you to roll a ball on a ramp to see the effect of gravity on it. This activity may be done by a group. Document the process of the activity with pictures and/or videos. marking the path of the ball and analyzing the resulting function is required.

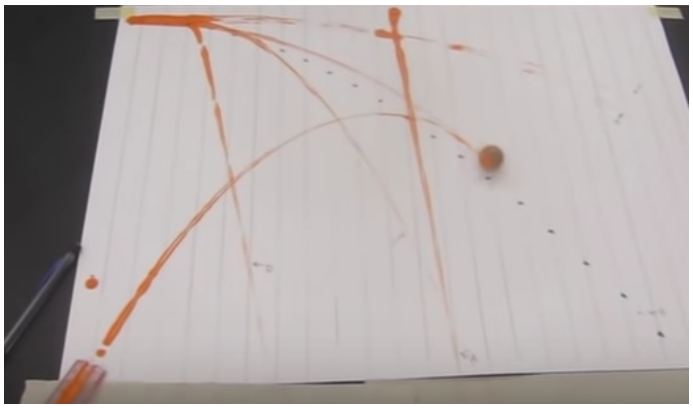
2. The second real-world activity asks you to seek out and interview a person within the community who uses math on a daily basis and understands how math relates to the real world outside the classroom. There are a series of questions for you to use when interviewing your individual. You will also need come up with three new questions to use. You will be asked to write a reflection paper after your interview. You may also earn bonus points by videoing your interview and/or creating a powerpoint presentation post-interview.

3. On your return to school you must document any and all materials and products that were created for the STEAM project. This may be done with pictures. Be prepared to present what you created in class.

Gravity and Parabolas

Calculus can be used to explain acceleration due to gravity. The rate of change of the position of an object, velocity, is the derivative of the position function. The rate of change of the velocity of an object, acceleration, is the derivative of the velocity function, the second derivative of the position function. This activity will show that acceleration due to gravity is a square relationship and will result in a curve that looks like a parabola. Because this activity requires a lot of material and work, it is better to do it together in a group. See these videos https://www.youtube.com/watch?v=4J_8IPwEcIc&t=8s and https://www.youtube.com/watch?v=in6tdnLgfK0 to understand what you will be doing.





Materials needed:

1. A large board
2. Paint
3. Marbles, ball bearings, or balls
4. Large sheets of paper or graph paper; alternatively, you may paint on the board directly and take pictures of the results before cleaning the paint off
5. A phone to take pictures and videos
6. Apparatus to start the ball (in the first video, it was a small ramp, in the second, the person rolled the ball)

Process: Slant the board, so that the ball will roll down due to gravity. Fasten the paper on the board, making sure that it is straight, so that the parabola will be easier to analyze. Construct and set up a ramp to give the ball a consistent start (a ruler with a groove in it with an eraser under one end was used in the first video). Get an idea of how a ball will travel on the board by rolling it several times, before painting the ball. Put paint on the ball and roll it again, so that its path will be painted on the paper. Put a coordinate system on the paper (this is easier if you are using graph paper). Document the process of the activity with pictures and/or videos. Record several of the points on the parabola. Use technology to find the equation of the parabola. The simplest way to do this is to perform a quadratic regression on a graphing calculator. You will turn in the graphs that your ball painted, your analysis of a graph, and pictures or videos of your project.

Community Project:

Real World Application Interview

Objective: to obtain an interview with a person within the community who uses math on a daily basis and understands how math relates to the real world outside the classroom.

How to go about the project:

1. Initiate an interview with a person within the community who uses math on a daily basis. (Try to find someone you do not have daily contact with such as a family member or friend. Get out of your comfort zone!) Schedule a date and time for the interview. Here are some examples of people you could interview, or find someone else who "uses math on a daily basis" in their chosen career.
    a. **Algebra – bank loan officer, accountant, engineer, chef, artist, pilot, salon owner**
    b. **Geometry – construction contractor, architect, landscape artist, baker, artist, pilot**
    c. **Algebra 2 – bank loan officer, financial advisor, engineer, accountant, pilot, statistician**
    d. **Calculus – engineer, financial advisor, aeronautics, statistician**
2. In addition to using the questions provided below to interview the professional, you must create three unique questions to ask your professional. They are worth points toward your grade. Think of things that you would like to find out about as well as what others may wish to know. These should be written **prior** to the interview.
3. Interview person of interest and take note of their responses.
    a. **EXTRA CREDIT OPPORTUNITIES: (choose one, not both!)**
        i. **Video record the interview to be turned in with your report. If you choose to record the interview, it must be between 5 and 10 minutes in length.**
        ii. **Create a PowerPoint presentation describing your interview.**
4. Once your interview is complete, write or type a one page summary/reflection over this experience. Your reflection should include: why you chose this person to interview, the questions asked with responses, information you learned through this interview, and explanation of whether or not this career interests you, how it applies to something you have learned in math class, etc.

Name_____ Class_____

Person Interviewing_____Occupation_____

Date of Interview_____ Location_____

1.) Reason you chose this person to interview:

2.) What is your occupation?

3.) Where do you work and how long have you worked there?

4.) Is a degree required to apply for this position? If so, what is the degree?

5.) What skills must you have in order to do your job?

6.) How do you use math on a daily basis? What type of math do you use?

7.) What do you like most about your job? What do you like the least about your job?

8.) Would you recommend this occupation to others? Explain.

9.)

10.)

11.)